# Table of contents

# 8 Integrating Learning and Planning (MCTS)

# Model-Based Reinforcement Learning

*Last Module*: MDPs

*This Module*: learn model directly from experience

- . . . and use planning to construct a value function or policy

Integrates learning and planning into a single architecture

# Model-Based Reinforcement Learning

# Model-Based and Model-Free RL

Model-Free RL

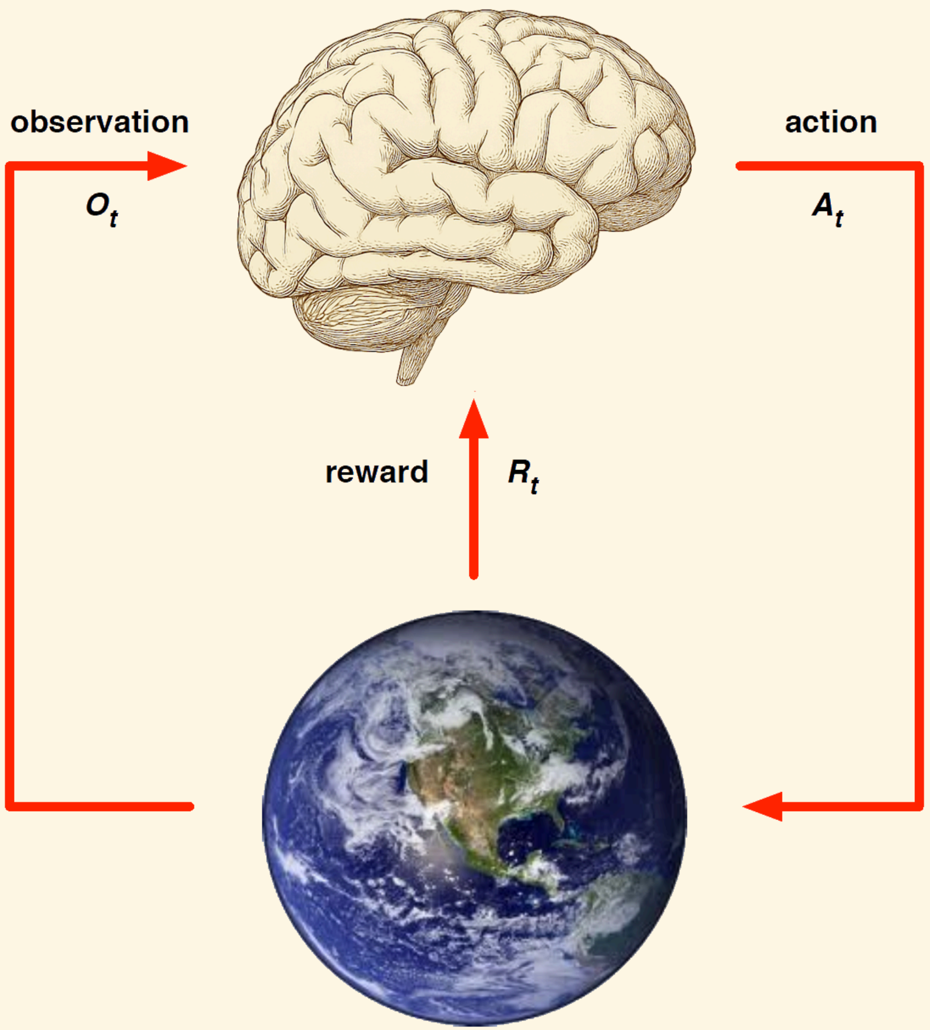
- No model
- Learn value function (and/or policy) from experience

Model-Based RL

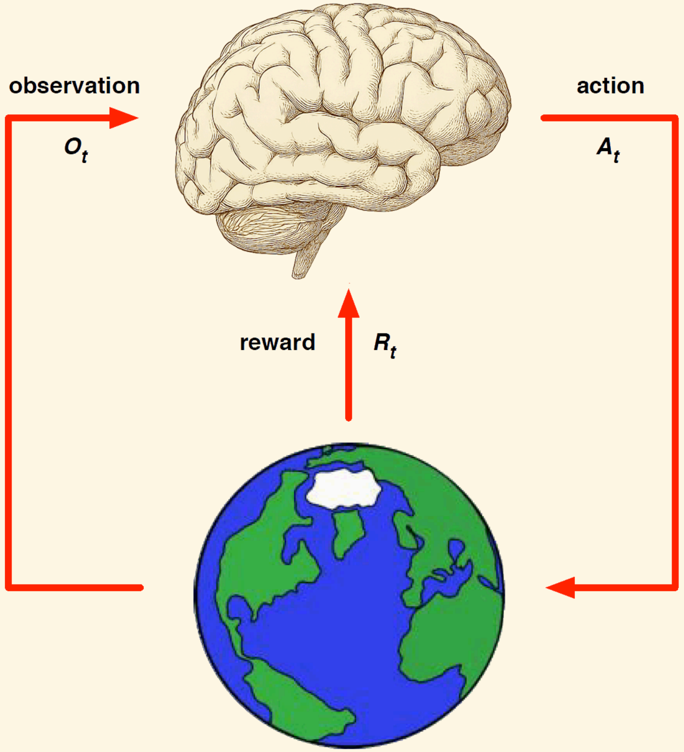
- Learn a model from experience
- Plan value function (and/or policy) from model

Lookahead by planning (*or thinking*) about what the value function will be

# Model-Free RL



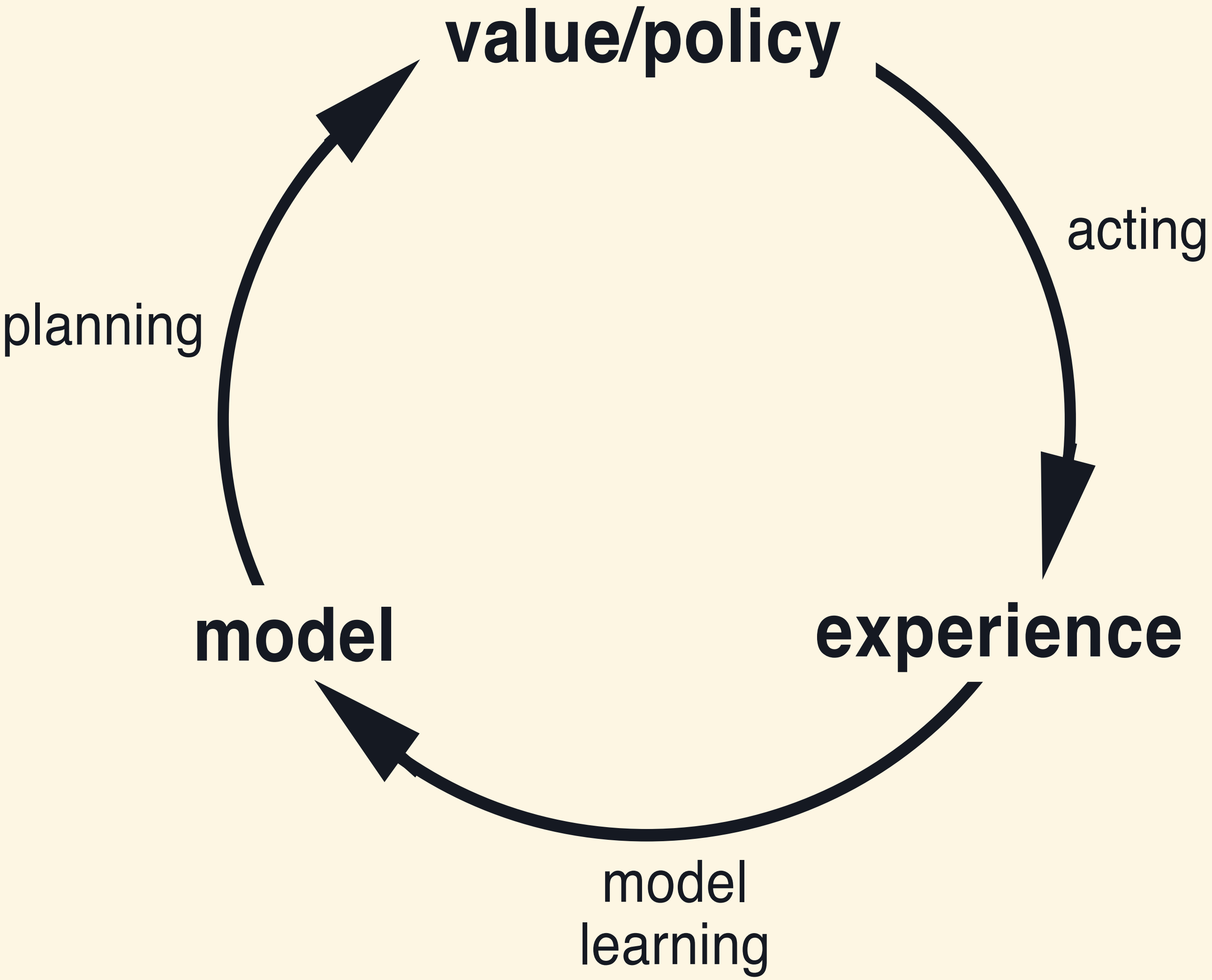
# Model-Based RL




Replace real world with the agent's (simulated) *model* of the environment

- Supports rollouts (lookaheads) under imagined actions to reason about what value function will be, without further environment interaction

# Model-Based RL (2)



# Advantages of Model-Based RL

Advantages:

- Can efficiently learn model by supervised learning methods
- Can reason about model uncertainty, and even take actions to reduce uncertainty

Disadvantages:

- First learn a model, then construct a value function

  $\Rightarrow$ two sources of approximation error

# Learning a Model

# What is a Model?

A *model* $\mathcal{M}$ is a representation of an MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$, *parameterised* by $\eta$

- We will assume state space $\mathcal{S}$ and action space $\mathcal{A}$ are known

So a model $\mathcal{M} = \langle \mathcal{P}_\eta, \mathcal{R}_\eta \rangle$ represents state transitions $\mathcal{P}_\eta \approx \mathcal{P}$ and rewards $\mathcal{R}_\eta \approx \mathcal{R}$

$$S_{t+1} \sim \mathcal{P}_\eta(S_{t+1} \mid S_t, A_t)$$

$$R_{t+1} = \mathcal{R}_\eta(R_{t+1} \mid S_t, A_t)$$

Typically assume conditional independence between state transitions and rewards

$$\mathbb{P}[S_{t+1}, R_{t+1} \mid S_t, A_t] \; = \; \mathbb{P}[S_{t+1} \mid S_t, A_t] \, \mathbb{P}[R_{t+1} \mid S_t, A_t]$$

Note you can learn from each (one-step) transition, treating the following step as the supervisor for the prior step.

# Model Learning

Goal: estimate model $\mathcal{M}_\eta$ from experience $\{S_1, A_1, R_2, \ldots, S_T\}$

This is a supervised learning problem

$$\begin{aligned} S_1, A_1 &\rightarrow R_2, S_2 \\ S_2, A_2 &\rightarrow R_3, S_3 \\ &\vdots \\ S_{T-1}, A_{T-1} &\rightarrow R_T, S_T \end{aligned}$$

- Learning $s, a \rightarrow r$ is a *regression* problem
- Learning $s, a \rightarrow s'$ is a *density estimation* problem
- Pick loss function, e.g. mean-squared error, KL divergence, ...
- Find parameters $\eta$ that minimise empirical loss

# Examples of Models

- Table Lookup Model
- Linear Expectation Model
- Linear Gaussian Model
- Gaussian Process Model
- Deep Belief Network Model
- $\cdots$ almost any supervised learning model

# Table Lookup Model

Model is an explicit MDP, $\hat{\mathcal{P}}, \hat{\mathcal{R}}$

- Count visits $N(s, a)$ to each state–action pair (parametric approach)

$$\hat{\mathcal{P}}^a_{s,s'} = \frac{1}{N(s,a)} \sum_{t=1}^{T} \mathbf{1}(S_t, A_t, S_{t+1} = s, a, s')$$

$$\hat{\mathcal{R}}^a_s = \frac{1}{N(s,a)} \sum_{t=1}^{T} \mathbf{1}(S_t, A_t = s, a)\, R_t$$

- Alternatively (a simple non-parametric approach)
  - At each time-step $t$, record experience tuple $\langle S_t, A_t, R_{t+1}, S_{t+1} \rangle$
  - To sample model, randomly pick tuple matching $\langle s, a, \cdot, \cdot \rangle$

# AB Example (Revisited) - Building a Model

Two states $A, B$; no discounting; 8 episodes of experience

$A, 0, B, 0$

$B, 1$

$B, 1$

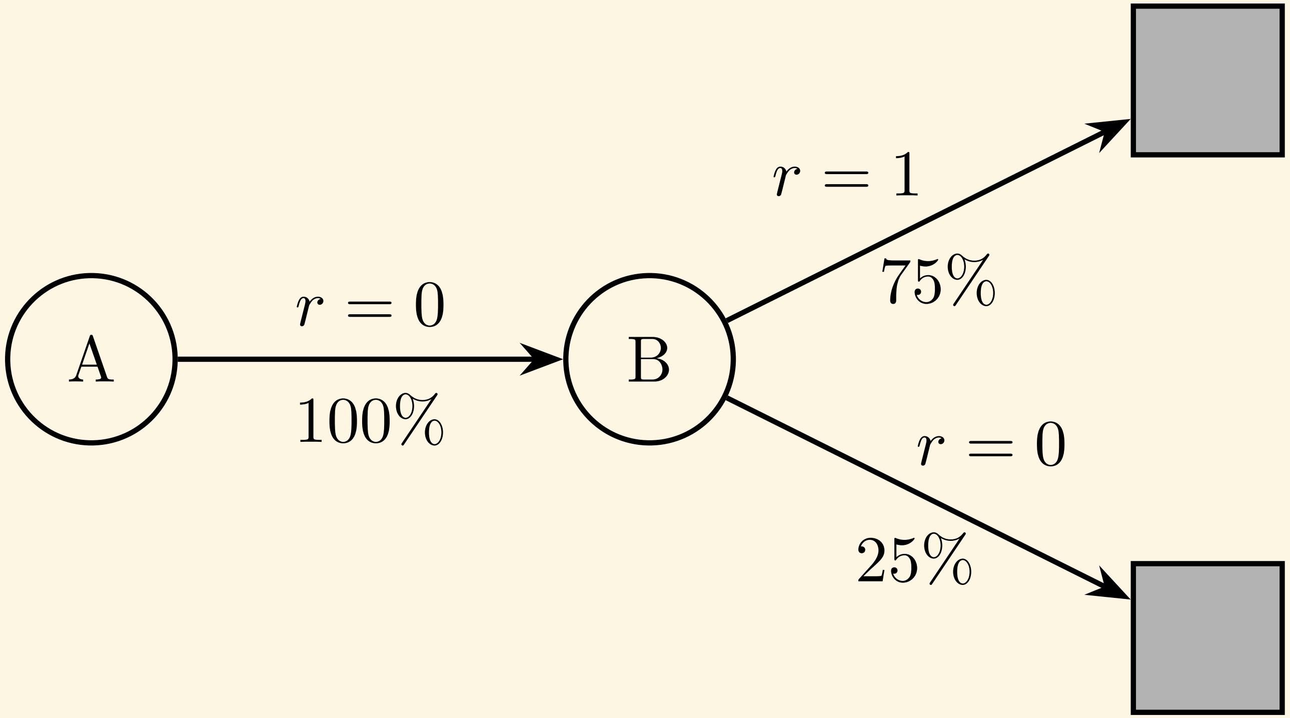
$B, 1$

$B, 1$

$B, 1$

$B, 1$

$B, 0$




We have constructed a table lookup model from the experience

# Planning with a Model

# Planning with a Model

Given a model $\mathcal{M}_\eta = \langle \mathcal{P}_\eta, \mathcal{R}_\eta \rangle$

- Solve the MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}_\eta, \mathcal{R}_\eta \rangle$

Using favourite planning algorithm

- Value iteration
- Policy iteration
- Tree search
- $\ldots$

# Sample-Based Planning

A simple but powerful approach to planning is to use the model *only* to generate samples

Sample experience from model

$$S_{t+1} \sim \mathcal{P}_\eta(S_{t+1} \mid S_t, A_t)$$
$$R_{t+1} = \mathcal{R}_\eta(R_{t+1} \mid S_t, A_t)$$

Apply *model-free* RL to samples, e.g.:

- Q-learning
- Monte-Carlo control or Sarsa

Sample-based planning methods are often more efficient

# Back to the AB Example

- Construct a table-lookup model from real experience
- Apply model-free RL to sampled experience

Real experience

$A, 0, B, 0$

$B, 1$

$B, 1$

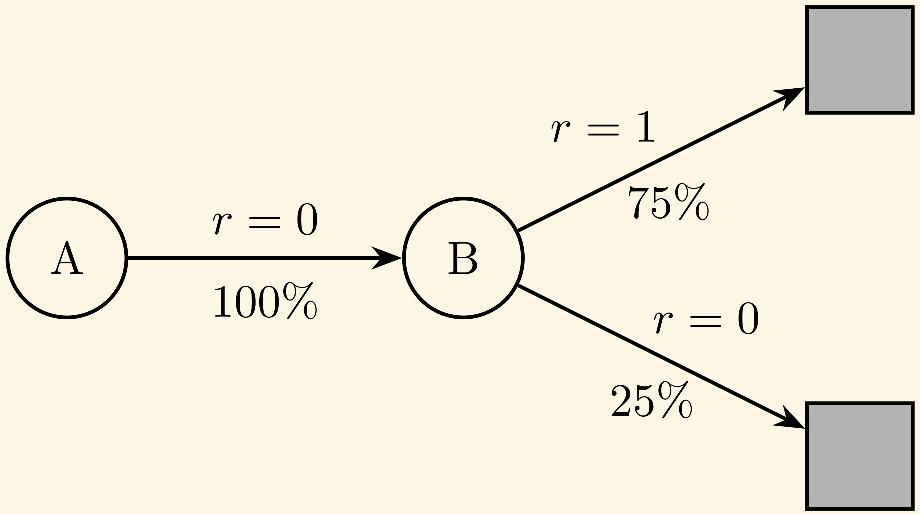
$B, 1$

$B, 1$

$B, 1$

$B, 1$

$B, 0$




Sampled experience

$B, 1$

$B, 0$

$B, 1$

$A, 0, B, 1$

$B, 1$

$A, 0, B, 1$

$B, 1$

$B, 0$

e.g. Monte-Carlo learning: $V(A) = 1; V(B) = 0.75$

We can sample as many trajectories as we want from the model, unlike from the environment

- we essentially have infinite data

# Planning with an Inaccurate Model

Given an imperfect model $\langle \mathcal{P}_\eta, \mathcal{R}_\eta \rangle \neq \langle \mathcal{P}, \mathcal{R} \rangle$

- Performance of model-based RL is limited to optimal policy for approximate MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}_\eta, \mathcal{R}_\eta \rangle$
- i.e. Model-based RL is only as good as the estimated model

When the model is *inaccurate*, planning process will compute a sub-optimal policy

- **Solution 1:** when model is wrong, use model-free RL
- **Solution 2:** reason explicitly about model uncertainty (e.g. Bayesian approach)

# Real and Simulated Experience

We consider two sources of experience

**Real experience** Sampled from environment (true MDP)

$$S' \sim \mathcal{P}_{s,s'}^{a}$$
$$R = \mathcal{R}_{s}^{a}$$

**Simulated experience** Sampled from model (approximate MDP)

$$S' \sim \mathcal{P}_{\eta}(S' \mid S, A)$$
$$R = \mathcal{R}_{\eta}(R \mid S, A)$$

# Integrating Learning and Planning

Model-Free RL

- No model
- Learn value function (and/or policy) from real experience

Model-Based RL (using Sample-Based Planning)

- Learn a model from real experience
- Plan value function (and/or policy) from simulated experience

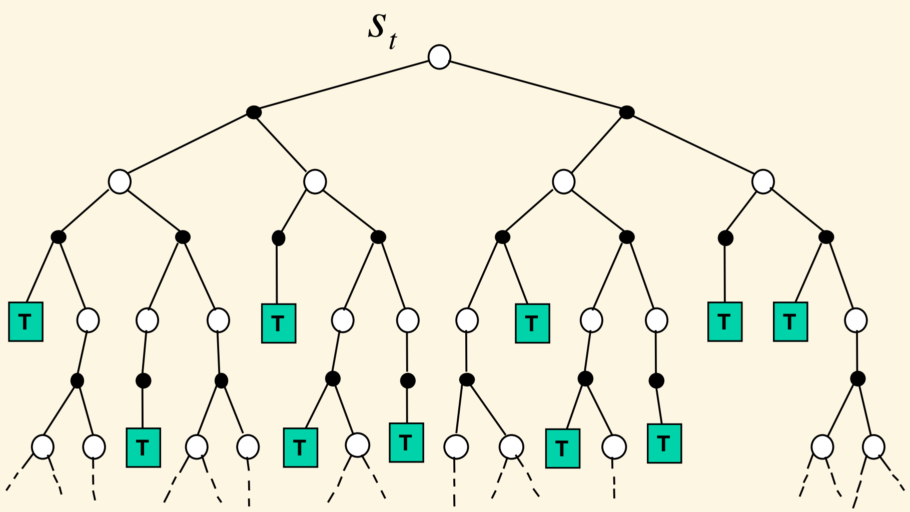
## Integrated Architectures

- Learn a model from real experience
- Learn and plan value function (and/or policy) from real *and* simulated experience

# Simulation-Based Search

# Simulation-Based Search (1)

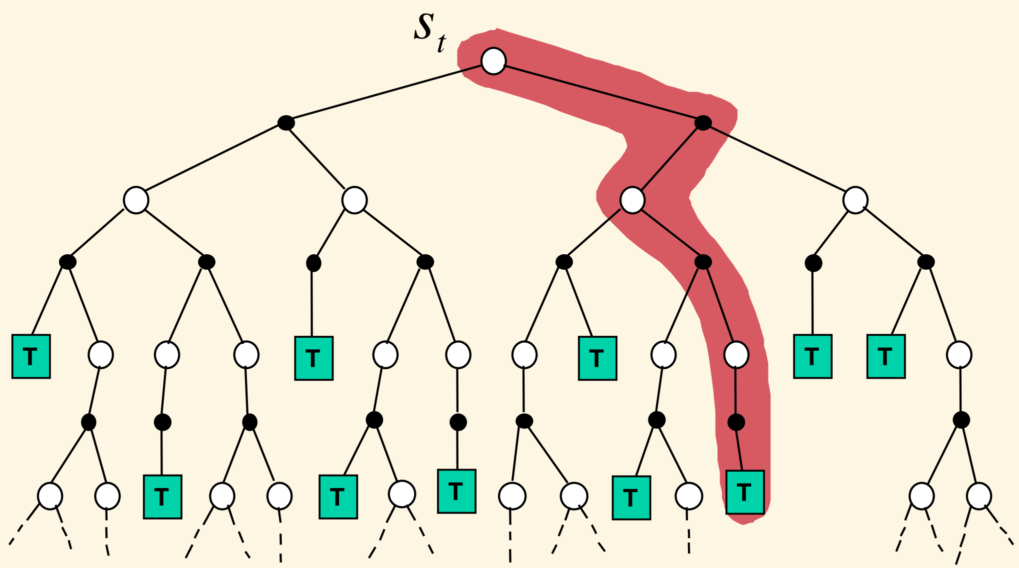
- Forward search algorithms select the best action by look-ahead
- They build a search tree with the current state $s_t$ at the root
- Using a model of the MDP to look ahead




No need to solve whole MDP, just sub-MDP starting from now

# Simulation-Based Search (2)

- Forward search paradigm using sample-based planning
- Simulate episodes of *experience* from now with the model
- Apply model-free RL to simulated episodes



# Simulation-Based Search (3)

Simulate episodes of experience from now with the model

$$\{s_t^k, A_t^k, R_{t+1}^k, \ldots, S_T^k\}_{k=1}^K \sim \mathcal{M}_\nu$$

Apply model-free RL to simulated episodes

- Monte-Carlo control $\rightarrow$ Monte-Carlo search
- Sarsa $\rightarrow$ TD search

# Simple Monte-Carlo Search

Given a model $\mathcal{M}_\nu$, and a simulation policy $\pi$

- For each action $a \in \mathcal{A}$
  - Simulate $K$ episodes from current (real) state $s_t$

$$\{s_t, a, R^k_{t+1}, S^k_{t+1}, A^k_{t+1}, \ldots, S^k_T\}^K_{k=1} \sim \mathcal{M}_\nu, \pi$$

- Evaluate actions by mean return (Monte-Carlo evaluation)

$$Q(\boldsymbol{s_t}, \boldsymbol{a}) = \frac{1}{K} \sum_{k=1}^{K} G_t \xrightarrow{P} q_\pi(s_t, a)$$

- Select current (real) action with maximum value $a_t = \arg\max_{a \in \mathcal{A}} Q(s_t, a)$

# Monte-Carlo Tree Search (Evaluation)

Given a model $\mathcal{M}_\nu$, simulate $K$ episodes from current state $s_t$ using current simulation policy $\pi$

$$\{s_t, A_t^k, R_{t+1}^k, S_{t+1}^k, \ldots, S_T^k\}_{k=1}^K \sim \mathcal{M}_\nu, \pi$$

- Build a search tree containing visited states and actions
- Evaluate states $Q(s, a)$ by *mean* return of episodes from each pair $s, a$

$$Q(s, a) = \frac{1}{N(s, a)} \sum_{k=1}^{K} \sum_{u=t}^{T} \mathbf{1}(S_u, A_u = s, a) G_u \overset{P}{\to} q_\pi(s, a)$$

After search is finished, select current (real) action with maximum value in search tree $a_t = \arg\max_{a \in \mathcal{A}} Q(s_t, a)$

# Monte-Carlo Tree Search (Simulation)

In MCTS, the simulation policy $\pi$ improves

Each simulation consists of two phases (in-tree, out-of-tree)

- Tree policy (improves): pick actions to maximise $Q(S, A)$
- Default policy (fixed): pick actions randomly

**Monte-Carlo Tree Search**

Repeat (each simulation)
Evaluate states $Q(S, A)$ by Monte-Carlo evaluation
Improve tree policy, e.g. by $\epsilon$–greedy($Q$)

Monte-Carlo control applied to simulated experience

- Converges on the optimal search tree,
  $Q(S, A) \rightarrow q_*(S, A)$

# Example: The Game of Go

# Example: The Game of Go

The ancient oriental game of Go is $2,500$ years old

Considered to be the hardest classic board game

Considered a grand challenge task for AI (*John McCarthy*)

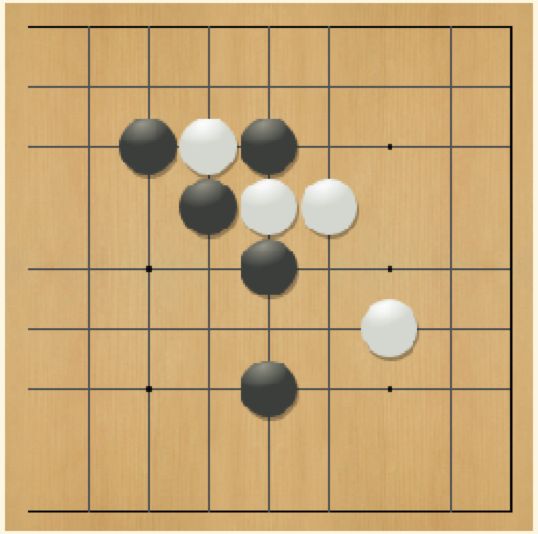
Traditional game-tree search has failed in Go

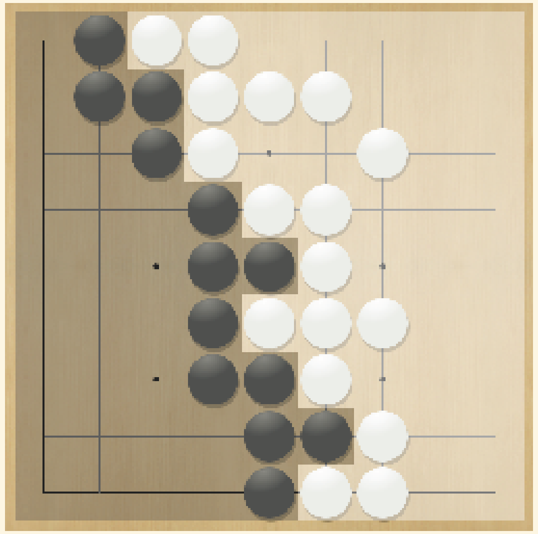
# Rules of Go

Usually played on 19x19, also 13x13 or 9x9 board

Simple rules, complex strategy

- Black and white place down stones alternately
- Surrounded stones are captured and removed
- The player with more territory wins the game





# Position Evaluation in Go

How good is a position $s$?

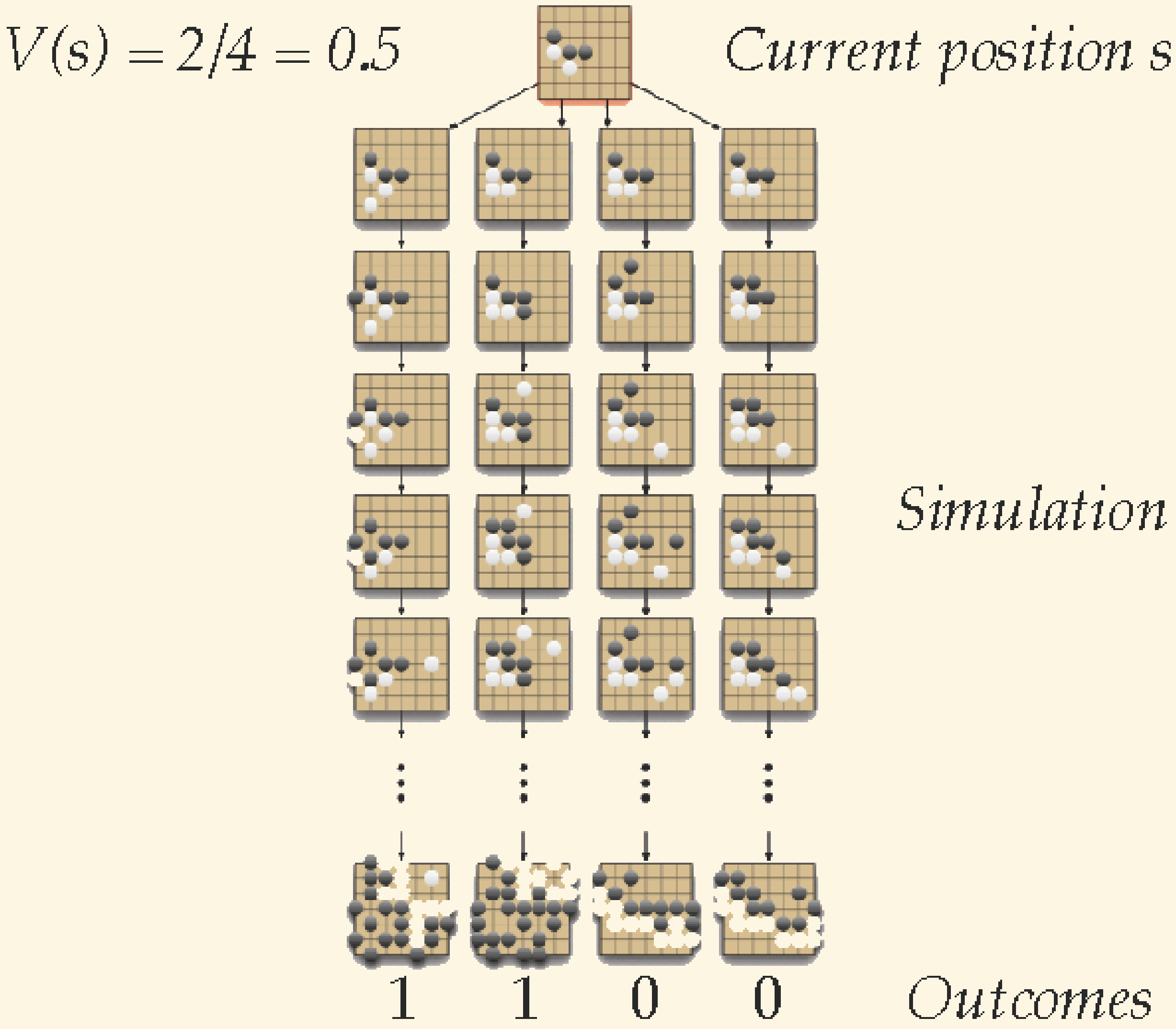
- Reward function (undiscounted):

$$R_t = 0 \quad \text{for all non-terminal steps } t < T$$
$$R_T = \begin{cases} 1 & \text{if Black wins} \\ 0 & \text{if White wins} \end{cases}$$

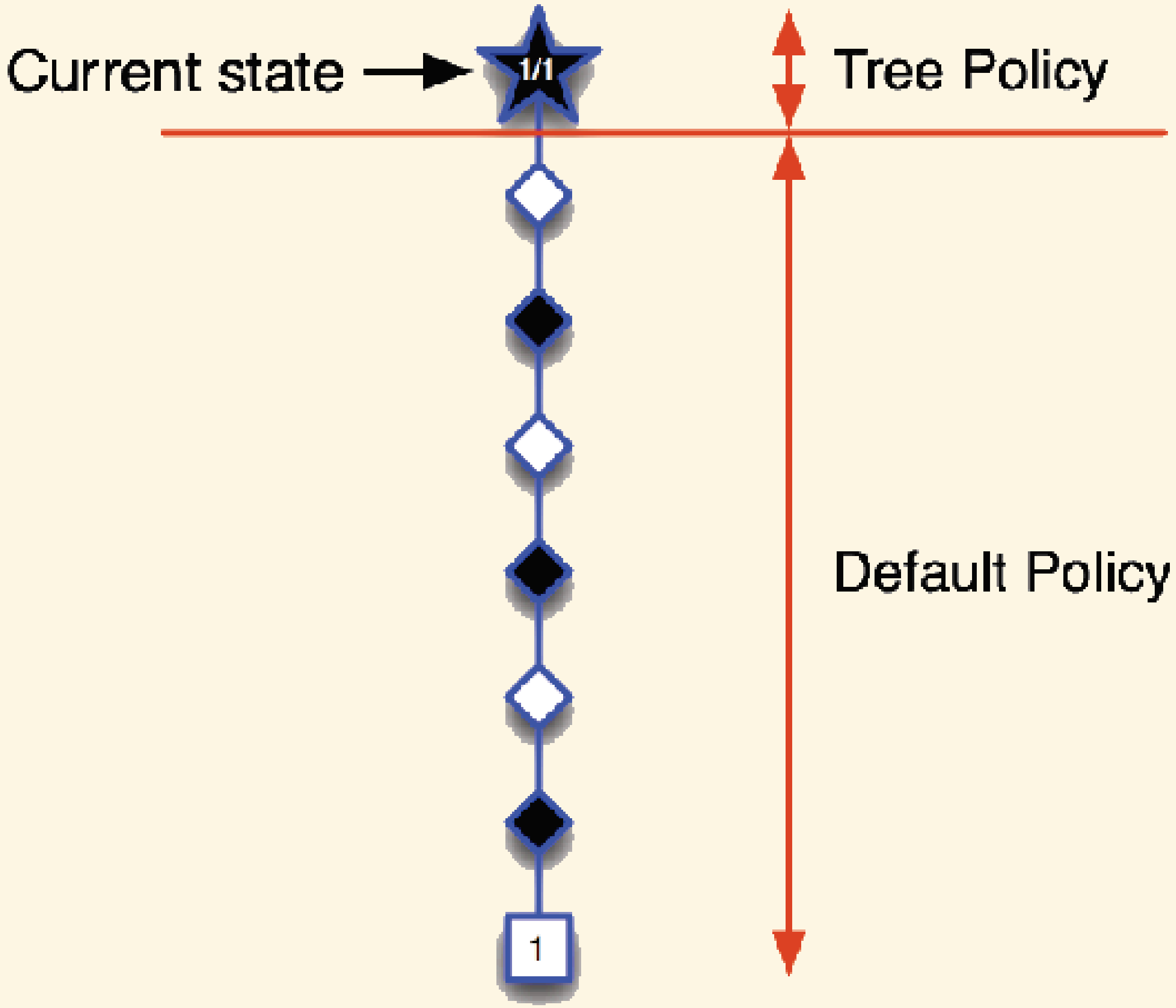
- Policy $\pi = \langle \pi_B, \pi_W \rangle$ selects moves for both players
- Value function (how good is position $s$):

$$v_\pi(s) = \mathbb{E}_\pi \left[ R_T \mid S = s \right] = \mathbb{P}[\text{Black wins} \mid S = s]$$
$$v_*(s) = \max_{\pi_B} \min_{\pi_W} v_\pi(s)$$

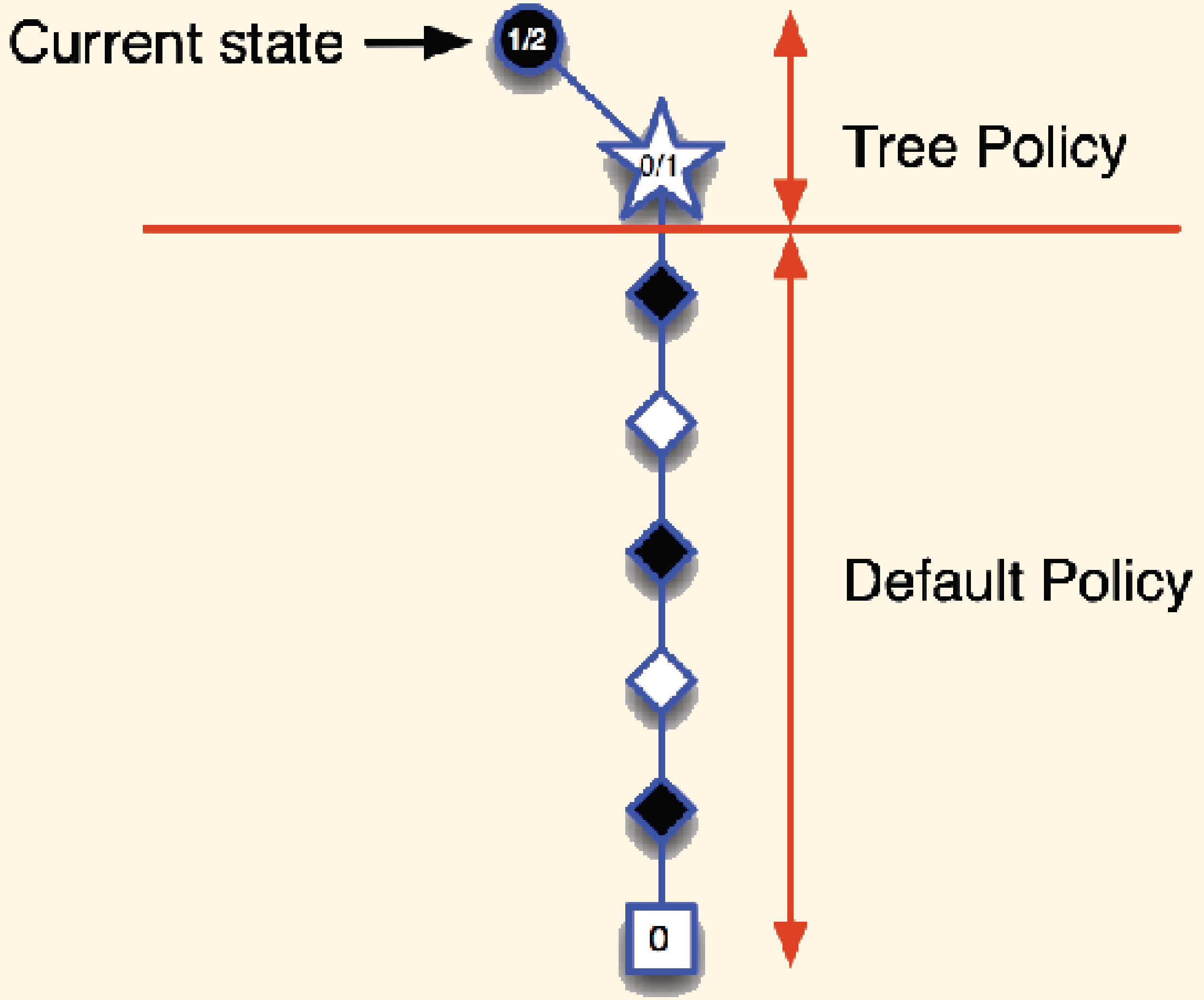
# Monte-Carlo Evaluation in Go



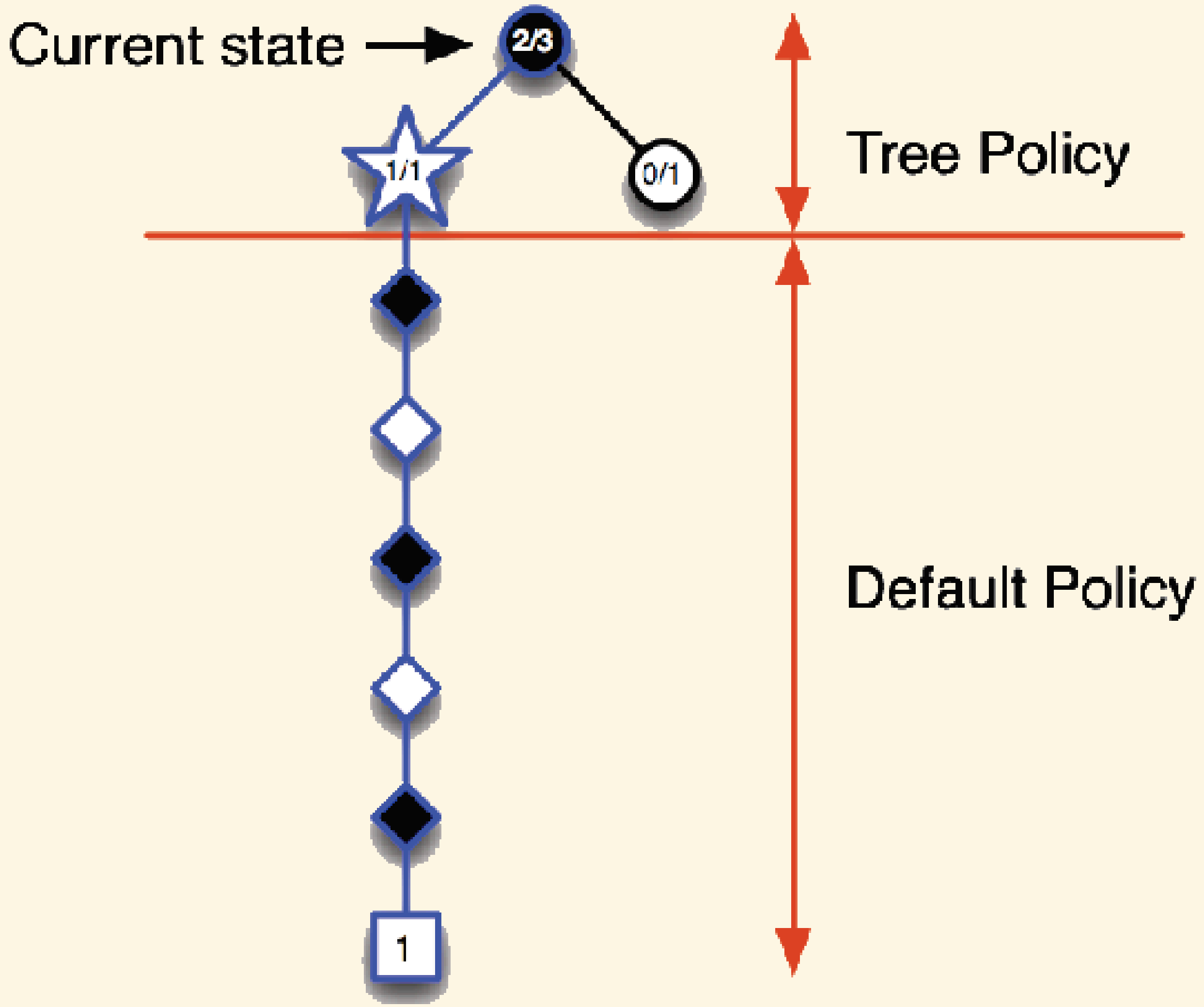
# Applying Monte-Carlo Tree Search (1)



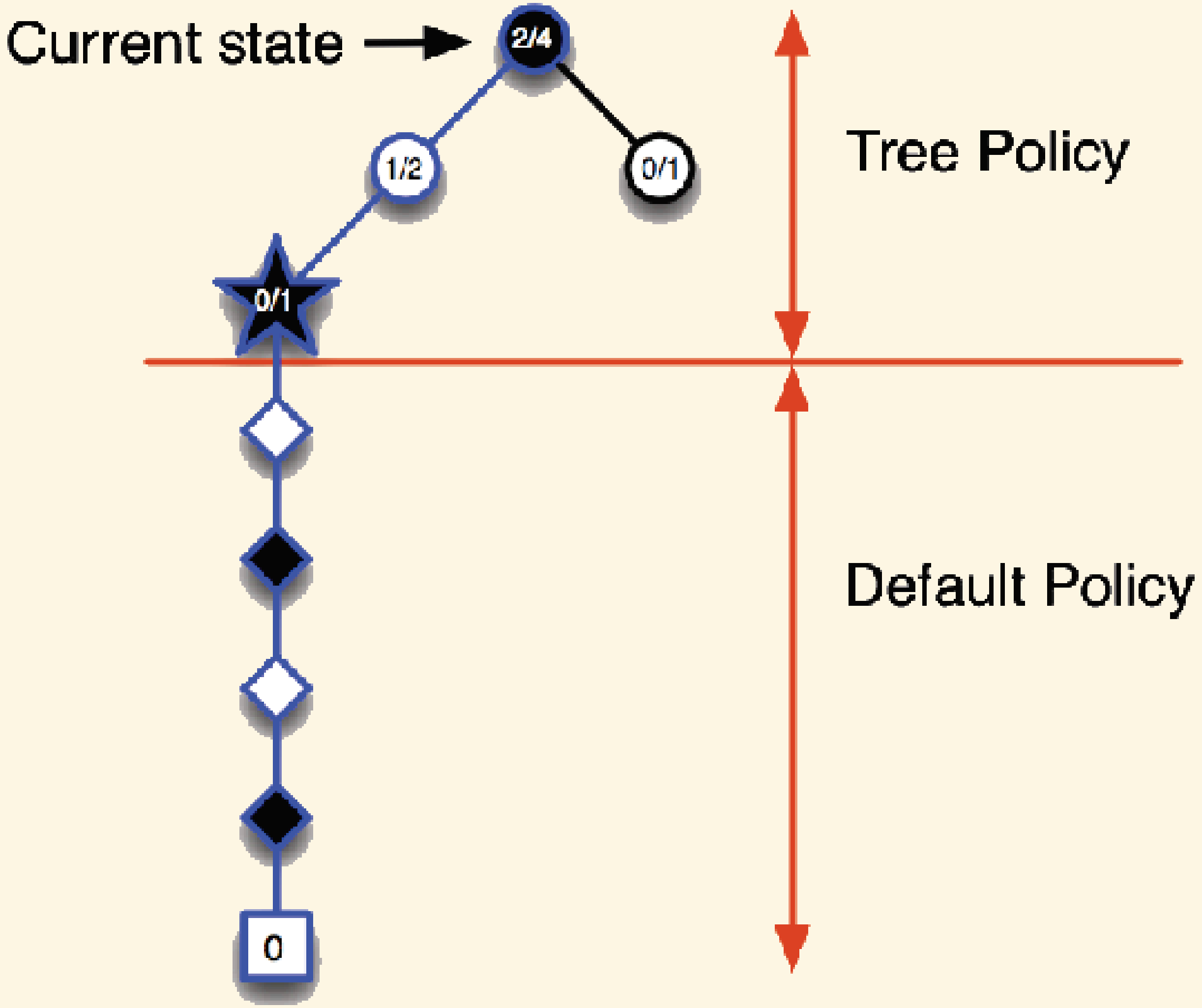
# Applying Monte-Carlo Tree Search (2)



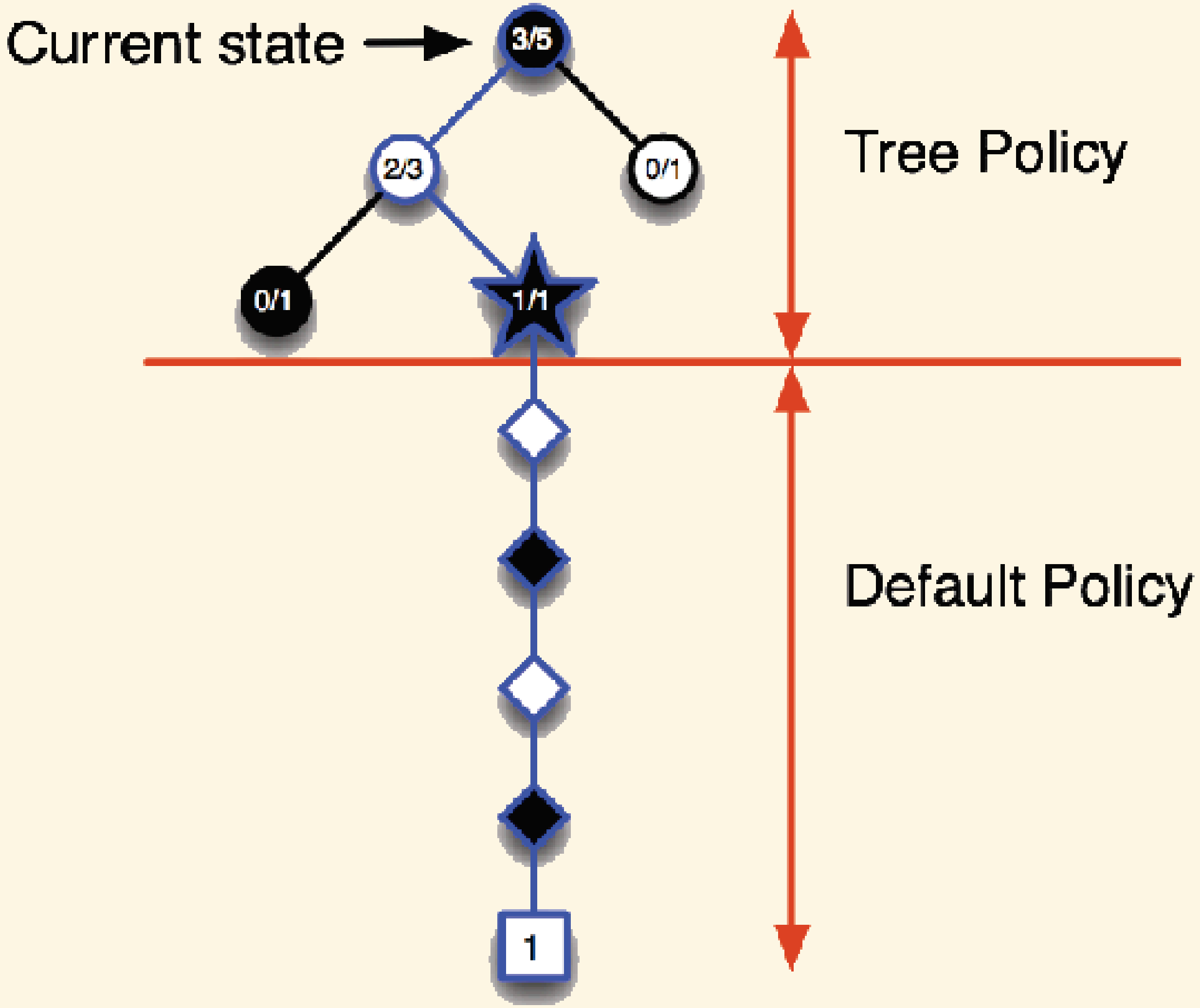
# Applying Monte-Carlo Tree Search (3)



# Applying Monte-Carlo Tree Search (4)



# Applying Monte-Carlo Tree Search (5)



# Advantages of MC Tree Search

Highly selective best-first search

- Evaluates states *dynamically* (unlike e.g. dynamic programming which does not use trees)
- Uses sampling to break curse of dimensionality

Works for “black-box” models (only requires samples)

- Computationally efficient, anytime, parallelisable